Multi-Unit Differential Auction-Barter Model for Electronic Marketplaces

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Abstract

Differential Auction-Barter (DAB) model augments the well-known double auction (DA) model with barter bids so that besides the usual purchase and sale activities, bidders can also carry out direct bartering of items. The DAB model also provides a mechanism for making or receiving a differential money payment as part of the direct bartering of items, hence, allowing bartering of different valued items. In this paper, we propose an extension to the DAB model, called the multi-unit differential auction-barter (MUDAB) model for e-marketplaces in which multiple instances of commodities are exchanged. Furthermore, a more powerful and flexible bidding language is designed which allows bidders to express their complex preferences of purchase, sell and exchange requests, and hence increases the allocative efficiency of the market compared to the DAB. The winner determination problem of the MUDAB model is formally defined, and a fast polynomial-time network flow based algorithm is proposed for solving the problem. The fast performance of the algorithm is also demonstrated on various test cases containing up to one million bids. Thus, the proposed model can be used in large-scale online auctions without worrying about the running times of the solver.

Keywords : Auction, Barter, Double Auction, E-marketplace, Multi Unit Differential Auction Barter

1 Introduction

The *double auction* (DA) institution allows market participants to submit asks (sale bids) and bids (purchase bids) for well-defined commodities, financial instruments or services. Beginning with Smith [1], many laboratory experiments with DA rules have been carried out (see [2] and [3] for a survey). The experiments demonstrate that the DA institution results in high allocative efficiency even with a small number of traders [4, 5, 6] which also explains the wide usage of the DA institution in the exchanges. The popularity of the institution has motivated researchers to propose and study variants of the DA. The *single-unit DA* is the base model in which each commodity in the market is considered as unique item. The *multi-unit DA* [7, 8] is an extension to the single unit DA which allows multiple instances of a commodity to be traded in the auction. Kalagnanam et al. [9] introduce multi-unit DA with assignment constraints and propose a network flow algorithm for finding the optimum allocation of commodities. In the *(multi-unit) combinatorial double auction* [10, 11], the institution further allows package (combinatorial) bidding in which the participants can submit bids on bundles of items instead of a single item. For an introductory survey of the DA, the reader is referred to [12].

An *electronic marketplace (e-marketplace)* is an electronic exchange that brings buyers and sellers together providing necessary regulations and services for trading. E-marketplaces offer services such as directory listings and searching of goods or services and transaction support. Secure, fast and reliable communication mediums in e-marketplaces help their customers in making new trading partnerships and reducing communication costs. However, for increasing the trading volume in the e-marketplaces, besides these services, a matching/brokering service is necessary between sellers and buyers in order to satisfy their possibly complex supply and demand requirements [13]. Auction based approaches have been proposed for this task [14]. For instance, Özturan [15] proposes a hybrid differential auction-barter (DAB) model which extends the single-unit DA institution for increasing the trading volume of online used vehicle auctions such as autobytel.com, the eBay Motors and the Yahoo! Autos. In these auctions, buyers place bids for the vehicles they want to purchase and sellers place asks for the cars they want to sell. Although bidders can submit both sale and purchase bids, these bids are considered as independent and they are not allowed to put conditional restrictions for their bids. This discourages bidders who are willing to sell their current vehicles only if they are able to purchase other vehicles. For instance, assume that a bidder currently owns a car of brand A and wants to exchange (barter) his car for a car of brand B while paying at most \$5000. He must first determine reservation prices for both car A and car B. The *reservation price* is the minimum (maximum) price at which the bidder is willing to sell (buy) an item. Assuming that the reservation price of the bidder for his car A is \$10,000, he will place an ask of \$10,000 for his car A and a bid of $\$10,000 + \$5,000 = \$15,000$ for car B. Depending on the auction outcome, he may lose his car without getting a replacement car or be forced to pay \$15,000 for an additional vehicle without selling his current vehicle, resulting in a \$10,000 budget deficit. In order to prevent this inefficiency, the DAB model extends the single-unit DA so that the bidders are also allowed to place *barter bids* in which an item can be bartered for another item. Besides, if the values of the items are not considered to be same, then the bidders are also allowed to declare an amount of money, called *differential price*, to be given or taken along with their barter request. For instance, for the above case, the bidder can submit a bid declaring that he wants to exchange his car A for car B and pay \$5000. This bid is represented using the following notation:

{Car A + \$5,000} => {Car B}

Since the DAB model is designed especially for online used car auctions (consumer-to-consumer), it is very unlikely to have more than one unit of an item (e.g. two identical used cars). Therefore, it is designed as a single-unit auction model and does not support multiple identical units of an item. The bidding language is also quite simple. A barter bid consists of exactly two items, an item to be given and an item to be taken, and an associated differential price if any. Similarly, a sale bid or a purchase bid consists of a single item and an associated price the bidder is willing to pay or earn.

In this paper, we propose a new model called the *multi-unit differential auction-barter* (MUDAB) model for e-markets in which multiple instances of commodities are exchanged. The model has been primarily designed for improving the efficiency of a clearinghouse [12] which is also known as call-market [16] or the discrete-time DA institution. In a *clearinghouse*, traders submits asks and bids during a predefined trading period, and at the end of this period, the market is cleared by matching the asks with the bids.

The model has three important features which are also explained quantitatively based on an example market scenario in the next section:

(*i*) The model extends both the multi-unit DA and the DAB models so that bidders can put forward barter bids along with the sale and purchase bids for multiple instances of items. This mechanism encourages traders to participate in the e-markets without risking buying of new items unless they sell their items first or vice-versa. Furthermore, in addition to the direct matching of sale and purchase bids as in DA, the model increases the allocative efficiency of the market by extracting barter cycles and sale-barter-purchase chains of any length in the market. An example scenario for a barter cycle involving three bidders is as follows:

$$\underbrace{\text{Bidder 1}}_{\textit{Barter}} \xrightarrow{\text{gives item(s) to}} \underbrace{\text{Bidder 2}}_{\textit{Barter}} \xrightarrow{\text{gives item(s) to}} \underbrace{\text{Bidder 3}}_{\textit{Barter}} \xrightarrow{\text{gives item(s) to}} \text{Bidder 1}$$

An example scenario for sale-barter-purchase chain involving four bidders is given below:

$$\underbrace{\substack{\text{Bidder 1}\\ \text{(earns money)}}}_{\textit{Sale}} \xrightarrow{\text{gives item(s) to}} \underbrace{\text{Bidder 2}}_{\textit{Barter}} \xrightarrow{\text{gives item(s) to}} \underbrace{\text{Bidder 3}}_{\textit{Barter}} \xrightarrow{\text{gives item(s) to}} \underbrace{\substack{\text{Bidder 4}\\ \text{(pays money)}}}_{\textit{Purchase}}$$

Finally, since the MUDAB model is the superset of both the multi-unit DA and the DAB models, the allocative efficiency of the MUDAB model is guaranteed to be at least as good as that of the multi-unit DA and the DAB models.

(*ii*) The model introduces a powerful and flexible bidding language (which is explained in Section 4 in detail) which allows bidders to express their complex preferences inside bids. Compared to DAB, the new bidding language allows bidders to combine purchase, sell and exchange requests inside the bids and put restrictions on the number of items to be traded. This feature improves the allocative efficiency of market, and also makes the MUDAB model preferable over the DAB model even for single-unit markets. Furthermore, the new bidding language reduces the total number of bids required for representing preferences of bidders greatly, and hence, reduces the amount of online information exchange between the bidders and the auctioneer.

(*iii*) The optimum allocation in the MUDAB model, that is the optimum set of satisfiable bids, can be determined using the proposed fast polynomial-time algorithm which is based on network flows [17]. Thus, the model can be used in large-scale web-based auctions containing millions of bids without worrying about the running times of the solver.

In the rest of the paper, we first compare the DA, DAB and MUDAB institutions quantitatively with an example scenario. Section 3 presents some electronic commerce applications that our model could be applied. In Section 4, a more comprehensive example with which we explain our MUDAB model is given. In Section 5, we formulate our model by using linear-integer programming and define the winner determination problem. In Section 6, we introduce a minimum cost network flow solution of the winner determination problem. Section 7 presents the experimental results and finally, the paper is concluded in Section 8.

2 Comparison of DA, DAB and MUDAB Models

In this section, the DA, DAB and MUDAB models will be compared based on an example scenario from a business-to-business paper market involving 5 bidders. The scenario is presented in Table 1a. In the paper industry, paper is manufactured in different qualities which we denote as grade A, B or C and is traded in standard sized rolls [9]. In this scenario, Bidder 1 wants to sell 200 rolls of his inventory, and Bidders 4 and 5 want to purchase 100 rolls of grade A and B paper respectively. Bidders 2 and 3, on the other hand, want to exchange (barter) their 100 rolls of inventory for lower grade paper. The reason could be that lower grade paper could satisfy their requirements and they would like to earn money by exchanging their inventory. The reservation prices of the bidders for different grades of paper are presented in Table 1b.

If the market was regulated using single-unit or multi-unit DA rules, then the first bidder would want to sell 200 rolls of grade B paper. The reason is that he considers the price of grade B paper higher than the price of grade C paper, and therefore, he would want to make more profit. Since there is no bartering option available in DA, the second and the third bidders either would not participate in the market because of the risk of losing their inventory and the risk of having a budget deficit, or they would take these risks and submit sale and purchase bids based on their subjective reservation prices. No decision making would be necessary for the fourth and the fifth bidders, and hence, they would simply submit purchase bids for the paper they want. As shown in Table 1c, the outcomes of the market would be same for both the single-unit and multi-unit cases. If the second and the third bidders do not participate, then only the sale bid of the first bidder would match the fifth bidder's purchase bid, causing a total of 100 rolls of paper to be traded. However, if they opt to participate, then the third bidder would have to sell 100 rolls to the fourth bidder without getting replacement paper and would have an inventory shortage. The single-unit and the multi-unit cases differ in the number of submitted bids. In first case, the bidders would have to submit one bid for each roll they want to sell and purchase. Therefore, the number of submitted bids in the single-unit case would be 400. However, in the latter case, one bid would be enough for each paper type, and hence, the bidders would have to submit only 3 bids.

If the market was a DAB institution, then the bidders would be able to place barter bids in addition to the sale and purchase bids. Therefore, the bidders 2 and 3 would submit barter bids in order to exchange their inventory for lower grade paper. The outcome of the market would be same as the DA case since there would be no barter cycles in the market. However, unlike the DA case, the barter bid mechanism would allow the bidders 2 and 3 to participate in the market without taking any risks. The downside of the DAB model is that the number of bids required for representing the preferences of the bidders is quite large. Since the model is single-unit, each roll of paper in the market would be introduced as a unique item. Therefore, one sale or purchase bid is required for each roll of paper to be sold or purchased as in the single-unit DA market. For the barter bids, this situation gets worse since the bidders should submit one barter bid for each pair of units of items they want to give and get. For instance, if a bidder wants to exchange m units of an item he owns for m units of another item of which there are n units available in the market, then he must place $m \cdot n$ barter bids. This scenario results in 40,600 bids.

In the MUDAB model, the bidding language allows more complex preferences to be declared. For instance, Bidder 1 can submit a single bid declaring that he wants to sell at most 200 rolls of his inventory without differentiating grade B or C paper. This feature of the model increases the outcome of the market. For this scenario, the MUDAB model finds the optimum allocation of goods which contains the following sale-barter-purchase chain of length 4 and sale-purchase match:

$$\underbrace{\substack{\text{Bidder 1}\\ \text{(earns \$22,000)}}}_{\textit{sale}} \xrightarrow[\text{grade C paper to}]{\text{gives 100 rolls}} \underbrace{\substack{\text{Bidder 2}\\ \left(\substack{\text{additionally}\\ \text{earns \$10,000}}\right)}}_{\textit{barter}} \xrightarrow[\text{grade B paper to}]{\text{gives 100 rolls}} \underbrace{\substack{\text{Bidder 3}\\ \left(\substack{\text{additionally}\\ \text{earns \$11,000}}\right)}}_{\textit{barter}} \xrightarrow[\text{grade A paper to}]{\text{gives 100 rolls}} \underbrace{\substack{\text{Bidder 4}\\ \text{(pays \$43,000)}}}_{\textit{purchase}}$$

$$\underbrace{\substack{\text{Bidder 1}\\ \text{(earns \$30,000)}}}_{\textit{sale}} \xrightarrow[\text{grade B paper to}]{\text{gives 100 rolls}} \underbrace{\substack{\text{Bidder 5}\\ \text{(pays \$30,000)}}}_{\textit{purchase}}$$

This allocation clearly satisfies all the requests of the bidders and no bidder pays more or earns less than the amount he declared as his reservation price. In this scenario, the MUDAB model requires only five bids to be submitted since the preference of each bidder can be represented with exactly one bid.

From the computational point of view, the allocation for the single-unit and multi-unit DAs can be found easily (by sorting and matching in log-linear time). In the DAB model, the allocation is found by solving a minimum cost flow problem for the network constructed by the DAB algorithm. Although this problem can also be solved in polynomial-time, the size of the network constructed by the DAB algorithm for multi-unit markets is so large that it may not be possible to find the optimum solution even for small markets. For instance, the network constructed by the DAB model for this small example has 1,302 nodes and 41,200 arcs. However, although the optimum allocation in the MUDAB model is also found by solving the same polynomial-time network flow problem, the size of the networks constructed by the MUDAB algorithm for the multi-unit markets is much smaller than that of the DAB model. The size of the network for this scenario contains only 23 nodes and 35 arcs.

As a final note, we will present the taxonomy of barter models which is illustrated in Table 2. Although the MUDAB model supports multiple identical units of items, it does not support package (combinatorial) bidding, that is bidding on bundles of items. For instance, a bidder cannot offer to exchange a bundle of items A and B together for a bundle of item C and D. Both single-unit and multi-unit *multi-resource barter problems* which support package bidding are studied by Özturan in [18]. It is proven that both cases of the multi-resource (multi-item) barter problem are

NP-Hard. Therefore, the MUDAB model does not incorporate package bidding feature because of the difficulty in finding the optimum allocation computationally. Heuristic approaches also would not be preferred from the economics perspective since in this case, there would be no incentive for the bidders to declare their true utility values inside their bids unless the set of preferences of the bidders is restricted [19]. Being able to calculate the optimum solution in polynomial time opens the way for designing truthful mechanisms [20, 21].

3 Electronic Commerce Applications

The MUDAB model is designed particularly as a core component of a matching/brokering service for increasing the quality of service. Although the model is also applicable to small-scale and possibly non-web-based markets, improvements in the trading volume and the allocative efficiency compared to DA can only be observed as the number of barter bids, and thus the barter cycles and chains inside the market increase. For this reason, the primary application area of the model is e-marketplaces in which bidders countrywide or even worldwide can participate and large number of bids can be collected in a short time.

In general, the MUDAB model can be applied to any single-unit or multi-unit e-market in which DA rules are applicable. As an example, the model can be used in a business-to-business e-marketplace where standardized units of overstocked raw materials such as metals, lumber, chemicals, paper, glass, and seed are exchanged. Another non-trivial market example would be vehicle exchanges for car dealerships in which both *new* and used vehicles are traded. For instance, in the National Auto Auction Association's (NAAA) North American member auctions, unlike the consumer car markets, multiple units of both new and used vehicles such as business and government fleets, replaced rental fleets, repossessed vehicles by financial institutions, and off-lease returns and trade-in vehicles are traded. According to the NAAA's 2007 annual review report ([22]), more than 9.54 million vehicles were sold among the offered 16 million vehicles in 2007.

In addition to the commercial multi-unit e-markets, the model can also be used in commercial barter exchanges [23]. A *commercial barter exchange* is an e-platform for businesses to trade their excess business capacities and assets, i.e. goods and services, which are regulated by an intermediary. Examples are Merchants Barter Exchange (merchantsbarter.com), BarterXchange (barterxchange.com) and Turk Barter International A.S. (turkbarter.com). In these exchanges, generally variants of DA institution are utilized. However, instead of using a country's currency, a private currency (e.g. barter dollar) whose value is tied to the corresponding country's currency (e.g. dollar) is used. The MUDAB model can readily be applied to the barter exchanges that use private currency.

The last application domain to be introduced is the e-media exchanges. An *e-media exchange* is an e-marketplace for trading e-media files such as e-books, music, movies and video games between the Internet users. Currently, there are many online stores that sell e-media such as iTunes and Amazon. However, to the best of our knowledge, there is no market for bartering e-media files legally. We propose an e-marketplace in which a central intermediary keeps records of the ownership information (i.e. licenses for e-media files), conducts auctions and regulates transactions among the users. The e-media files purchased by the users could be either served online by the intermediary or downloaded to the users' devices using some kind of digital right protection system. When a user sells a file or exchanges for another file, the intermediary removes the associated license from the user's account or transfers it to the new owner. Since all the transactions including the items are electronic, large-scale auctions with participants from all over the world could be conducted. Therefore, the MUDAB model would be preferred in this kind of exchanges for its fast polynomial-time winner determination algorithm. One final note is that although it may seem like each user will own at most one license for himself, in these exchanges;

- some people may engage in the trading of these files for just making profit, and therefore may own and trade multiple licenses of a file;
- wholesale license suppliers selling multiple units of licenses may also participate.

Even if no user wants to own more than one license for a file, there would be multiple licenses for that file owned by different users in the exchange which would also prevent the original DAB model being used in this kind of e-markets.

4 A Comprehensive Example and the Details of the Multi-Unit Differential Auction Barter Model

In this section, we will first introduce the bidding language of the MUDAB model, and then explain the model with an example scenario. In the bidding language of the MUDAB model, a bid is represented as follows:

$$\{\underbrace{(\text{item, exchange limit, price}), \ldots}_{\textit{Ask Set}}\} \xrightarrow{\text{bid level limit}} \{\underbrace{(\text{item, exchange limit, price}), \ldots}_{\textit{Request Set}}\}$$

For each bid, there is a set of three tuples, called the *ask set*, shown on the left hand side of the arrow. This set indicates the items to be sold or given in the exchange. Likewise, the right hand side of the arrow contains a set of three tuples, called the *request set*, which indicates the items to be purchased or taken. Each triple in these sets consists of an item, an *item level upper exchange limit* which is the maximum number of units of the item to be traded (i.e. purchased, sold or exchanged) and a reservation price per unit of the item. There is a logical OR relationship between the items listed in both the ask and request sets. Thus, any subset of items listed in the ask set can be bartered for any subset of items listed in the request set. However, in the MUDAB model, item exchanges are one-to-one so that each unit of an item can be exchanged for one unit of another item. If needed, in order to limit the number of units of items to be traded for the bid, the bid may also have a *bid level upper exchange limit* which is shown immediately above the arrow. This limit value puts an upper bound on the number of items to be traded for the corresponding bid.

For a barter request, in addition to the items to be bartered, the bidder should also declare a price difference between these items if the reservation prices of the items are different. In the previous DAB model, this price difference is directly encoded inside the bids along with the items to be bartered. This scheme is suitable for the previous model since its bidding language does not allow bidders to declare more than one item in both sides of a bid. However, in the MUDAB model, the bidding language allows both the ask and request sets to contain more than one item. In this case, a differential price is required for each pair of the items to be bartered. For instance, assume that a bidder wants to exchange items A, B, C with items D, E, F. For this request he may declare the differential prices between these items as shown in Figure 1a. In this scheme, the number of prices to be declared is equal to the product of the size of the ask and the size of the request set. Since this number gets larger as the sizes of these sets increase, in the MUDAB model, instead of declaring differential prices, each bidder declares prices for the items they want to give and they want to take as shown in Figure 1b. The price differences between the declared prices of items reflect the differential prices. This scheme reduces the number of prices to be declared for a bid to the sum of the sizes of the ask and request sets. However, it should be noted that prices of the items declared by the bidders inside the bids are totally subjective and their actual values affect neither the satisfiability of the bids nor the amount money to be paid/earned by the bidders as long as the price differences between the items reflect the additional (differential) amounts that the bidders are willing to give or get. To support the realism of this bidding format, we interviewed used car sellers. When they barter cars, they in fact go through the process of estimating the prices of the car to be given and to be obtained. Then, they determine the price difference which they will offer as additional payment to be received or given. This real life observation is another motivation for using this pricing scheme.

The following example illustrates the details of the MUDAB model and the new bidding language. Assume that four different bidders participate in a market. At the beginning, Bidder 1 owns 50 units of item A and 30 units of item B. Bidder 2 owns 40 units of item C and 20 units of item D. Bidder 3 owns 20 units of item D and Bidder 4 does not have any item.

Figure 2 illustrates the example scenario and its mathematical representation using the new bidding language is given in Figure 3. Bid 5 is an example of a basic *barter bid.* In this bid, bidder 3 wants to exchange *up to* 20 units of item D that he owns for the same number of units of item A. The bidder considers the price of each unit of item D as \$60 and the price of each unit of item A as \$120. Assuming that this bid is satisfied, meaning x units of item D are exchanged for x units of item A where $1 \leq x \leq 20$, the amount of money that the bidder must pay for this bid, which is called the *bid payoff*, is $x \cdot \$(120 - 60 = 60)$. If the reservation price of item A was lower than that of item D, then the bid payoff would be negative meaning that the bidder would earn this amount of money if the bid was satisfied. Since there is only one tuple in both the exchange and the request sets, the upper limits for items D and A are unnecessary as long as they are equal or higher than the bid level upper exchange limit. However, these limits are included in order to keep the integrity of the representation of the bidding language. If an upper limit value is not necessary, and therefore not declared, then it can be simply represented with the positive infinity (or the largest positive number).

In order to support sale and purchase requests inside the bids, a special item named $MONEY$ is introduced in this model. For instance, bid 4 is an example of a *sale bid.* In this bid, bidder 2 wants to sell up to 20 units of item C. Since the model enforces one-to-one exchanges, the bidder declares that he wants to exchange up to 20 units of item C with the same number of units of item $MONEY$. Since the reservation price of item $MONEY$ is \$0, and assuming that 20 units are sold, the bid payoff would be $\$-4800$ ($= 20 \cdot (0 - 240)$) meaning that the bidder would earn \$4800. In the MUDAB model, each unit of an item to be sold should be exchanged for one unit of item $MONEY$, and likewise one unit of item $MONEY$ should be exchanged for each unit of an item to be purchased. The item level upper exchange limit for item $MONEY$ limits the number of units of item to be sold or purchased for that bid. Unless a bidder requests a further restriction for the total number of units of items to be purchased or sold in all of his bids, each bidder is assigned with an infinite number of units of item $MONEY$.

Bid 6 is an example of a *purchase bid.* In this bid, bidder 4 declares that he wants to purchase up to 30 units of item B and up to 20 units of item D but no more than 40 units of both items which is indicated by the bid level upper exchange limit.

Bid 3 is a more complicated example in which the bidder wants to exchange two different items C, D for items A, B. The upper limits on the number of units to be exchanged are 30, 20, 30 and 30 respectively. The maximum number of units allowed for the exchange is 40. Suppose that this bid is satisfied, and let x_C, x_D, x_A and x_B be the numbers of units of items C, D, A and B that are exchanged ($0 \leq x_C \leq 30$, $0 \leq x_D \leq 20$, $0 \leq x_A \leq 30$ and $0 \leq x_B \leq 30$). Then, the payoff of this bid is equal to $(x_A \cdot \$120 + x_B \cdot \$190) - (x_C \cdot \$260 + x_D \cdot \$60)$. It should be noted that the total number of items to be given should be equal to the total number of items to be taken, that is $(x_C + x_D = x_A + x_B \leq 40)$.

By introducing the special item $MONEY$, the model allows bidders to express more complex preferences of purchase, sell and exchange inside a single bid. For instance, in the first bid, the bidder wants to exchange up to 40 units of his inventory for the same number of units of item D and at the same time he wants to sell the rest of his inventory without differentiating which item in his inventory is exchanged or sold. The outcome of this bid can be such that the bidder exchanges 40 units of item A for 40 units of item D, and sells 10 units of item A and 30 units of item B. However, the outcome can also be that the bidder exchanges 10 units of item A and 30 units of item B for 40 units of item D, and sells 40 units of item A. In both cases the amount of money that the bidder earns, that is \$9,000, would not be different. In bid 2, on the other hand, the bidder wants to have up to 40 units of item C without differentiating the cases in which "he exchanges 30 units of item B for 30 units of item C, and purchases 10 units of item C", or "he exchanges 20 units of item B for 20 units of item C, and purchases 20 units of item C". The exchange limits in this bid also puts a limit on the amount of money that the bidder would have to pay which is \$(20 * 250 + 20 * 50 = 6000). It should be noted that although bidder 1 offers to give 30 units of item B in both his bids, bid 1 and 2, the model does not allow the bidder to give more than the number of units of items he owns in the final outcome.

In the given example, the reservation price of the item $MONEY$ is set to zero for the ease of understanding. However, the model also allows assigning reservation prices for the item $MONEY$ in order to increase the expressibility of the bids. For instance, assume that a bidder owns 10 instances of item A and requests 10 instances of item B. He considers the price difference between each unit of these items as \$100. Furthermore, if bartering would not be possible, then he wants to sell each unit of his item A for the price of \$80 or purchase each unit of item B for the price of \$160. This preference of the bidder can be expressed by the following bid:

$$\{(A, 10, \$100), (MONEY, 10, \$40)\} \xrightarrow{10} \{(B, 10, \$200), (MONEY, 10, \$20)\}$$

Here, it should be noted that if reservation prices other than 0 are assigned to the item $MONEY$, then the reservation price of the item $MONEY$ in the ask set cannot be lower than the reservation price of the item $MONEY$ in the request set. Otherwise, this situation may cause the bidder to pay money without getting any item.

5 Formulation of the MUDAB Model

The MUDAB model is formally defined as follows: Let $C = \{c_1, c_2, \ldots, c_m\}$ be the set of m bidders and $R = \{MONEY, r_2, r_3, \ldots, r_n\}$ be the set of n items in the market. The first item $MONEY(= r_1)$ represents the special item which is used for purchase and sale requests inside the bids. Let O be an m-by-n ownership matrix where element o_{ij} defines the number of units of item r_j owned by bidder c_i ($o_{ij} \in \mathbb{Z}^+ \cup 0$). We define B_i as the set of bids submitted by the bidder c_i and the set of all bids, B, is defined as $B = \bigcup_{i=1}^{m} B_i$. Each bid b_k is denoted by a triplet, $b_k = (A_k, u_k, R_k)$, where A_k is the ask set, u_k is the bid level upper exchange limit ($u_k \in \mathbb{Z}^+$) and R_k is the request set of the bid b_k. The ask set is used for declaring items to be bartered or sold in the market. It consists of y three tuples, $A_k = \{(i'_1, u'_1, p'_1), \ldots, (i'_y, u'_y, p'_y)\},\ y \in \mathbb{Z}^+$, and in each tuple (i'_t, u'_t, p'_t), i'_t denotes the index of the item to be bartered or sold, u'_t denotes the item level upper exchange limit, and p'_t is the reservation price of the item ($1 \leq t \leq y, r_{i'_t} \in R, u'_t \in \mathbb{Z}^+, p'_t \in \mathbb{R}$). The request set, on the other hand, is used for declaring items to be bartered for or purchased. It consists of z three tuples, $R_k = \{(i''_1, u''_1, p''_1), \ldots, (i''_z, u''_z, p''_z)\},\ z \in \mathbb{Z}^+$, and in each tuple (i''_v, u''_v, p''_v), i''_v denotes the index of the item to be taken or purchased, u''_v denotes the item level upper exchange limit, and p''_v is the reservation price of the item ($1 \leq v \leq z, r_{i''_v} \in R, u''_v \in \mathbb{Z}^+, p''_v \in \mathbb{R}$). In this definition, in order to prevent the confusion and increase the readability, the superscript ($'$) is used for the variables of the ask set and the superscript ($''$) is used for the variables of the request set.

A bid b_k is called *satisfiable* if and only if the following equalities hold:

$$x'_{kt} \leq u'_t \ (\forall t \,|\, (i'_t, u'_t, p'_t) \in A_k),$$

$$x''_{kv} \leq u''_v \ (\forall v \,|\, (i''_v, u''_v, p''_v) \in R_k) \ \text{ and}$$

$$1 \leq \sum_{\forall t \,|\, (i'_t, u'_t, p'_t) \in A_k} x'_{kt} = \sum_{\forall v \,|\, (i''_v, u''_v, p''_v) \in R_k} x''_{kv} \leq u_k.$$

where x'_{kt} denotes the number of units of item to be given by tth tuple of the ask set A_k of the bid b_k, and x''_{kv} denotes the number of units of items to be taken by vth tuple of the request set R_k of the bid b_k. The bid payoff (p_k) for the bid b_k is calculated as follows:

$$p_k = \sum_{\forall v \,|\, (i''_v, u''_v, p''_v) \in R_k} p''_v x''_{kv} - \sum_{\forall t \,|\, (i'_t, u'_t, p'_t) \in A_k} p'_t x'_{kt}$$

The *winner determination problem of the MUDAB model* is defined as finding a set of mutually satisfiable bids such that the sum of the payoffs of the bids is maximized. If the maximum sum of the bid payoffs is negative for a problem instance, then this means that there is no feasible solution unless the deficit is subsidized by the auctioneer.

It should be noted that the number of units of the item $MONEY$ owned by a bidder c_i, that is o_{i1}, defines an upper bound for the difference between the number of units of item that he can purchase and the number of units of item that he can sell in all of his bids. If such a restriction is not necessary for the bidder, then this value is set to the largest positive number.

The linear integer programming formulation of the winner determination problem is as follows:

$$\text{maximize} \sum_{\forall k \,|\, b_k \in B} \left(\sum_{\forall v \,|\, (i''_v, u''_v, p''_v) \in R_k} p''_v x''_{kv} - \sum_{\forall t \,|\, (i'_t, u'_t, p'_t) \in A_k} p'_t x'_{kt} \right) \quad (1)$$

$$\text{subject to} \sum_{\forall v \,|\, (i''_v, u''_v, p''_v) \in R_k} x''_{kv} - \sum_{\forall t \,|\, (i'_t, u'_t, p'_t) \in A_k} x'_{kt} = 0 \quad (\forall k \,|\, b_k \in B) \quad (2)$$

$$\sum_{\forall k, v \,|\, b_k \in B \,\wedge\, (i''_v, u''_v, p''_v) \in R_k \,\wedge\, i''_v = j} x''_{kv} - \sum_{\forall k, t \,|\, b_k \in B \,\wedge\, (i'_t, u'_t, p'_t) \in A_k \,\wedge\, i'_t = j} x'_{kt} = 0 \quad (\forall j \,|\, r_j \in R) \quad (3)$$

$$\sum_{\forall k, t \,|\, b_k \in B_i \,\wedge\, (i'_t, u'_t, p'_t) \in A_k \,\wedge\, i'_t = j} x'_{kt} \leq o_{ij} \quad (\forall i, j \,|\, c_i \in C, r_j \in R) \quad (4)$$

$$x'_{kt} \leq u'_t \quad (\forall k, t \,|\, b_k \in B \,\wedge\, (i'_t, u'_t, p'_t) \in A_k) \quad (5)$$

$$x''_{kv} \leq u''_v \quad (\forall k, v \,|\, b_k \in B \,\wedge\, (i''_v, u''_v, p''_v) \in R_k) \quad (6)$$

$$\sum_{\forall t \,|\, (i'_t, u'_t, p'_t) \in A_k} x'_{kt} \leq u_k \quad (\forall k \,|\, b_k \in B) \quad (7)$$

$$x'_{kt}, x''_{kv} \in \mathbb{Z}^+ \cup \{0\} \quad (\forall k, t, v) \quad (8)$$

In this formulation, the objective line in Eq.(1) maximizes the sum of the payoffs of all satisfied bids. Eq.(2) satisfies the one-to-one exchange requirement of the model such that for every bid the number of units of items to be taken must be equal to the number of units of items to be given. Eq.(3) imposes a basic feature of any market. For any bidder, in order to purchase or get an item in the exchange, there must be a bidder that sells or gives the same item. Therefore, for each item, the number of units of items taken by all bidders must be equal to the number of units of items given by all bidders. Furthermore, it is clear that for each item, no bidder can give more than the number of units he owns and this restriction is enforced with Eq.(4). Eq.(5) and Eq.(6) apply the item level upper bound restrictions and finally Eq.(7) ensures that for any bid, the number of units of items to be exchanged does not exceed the bid level upper exchange limit.

An Issue Related to the Unrequested Items

In the MUDAB model, bidders should work out and declare the price differences between the items they want to give and the items they want to get inside the bids. Since the reservation prices of the items are subjective, the following situation may occur. Consider the following bids:

$$\begin{aligned}
&\text{Bidder 1: } b_1 = \{(A, 1, \$230)\} \xrightarrow{1} \{(MONEY, 1, \$0)\}\\
&\text{Bidder 2: } b_2 = \{(B, 1, \$350)\} \xrightarrow{1} \{(A, 1, \$220)\}\\
&\text{Bidder 3: } b_3 = \{(C, 1, \$60)\} \xrightarrow{1} \{(B, 1, \$380)\}\\
&\text{Bidder 4: } b_4 = \{(D, 1, \$20)\} \xrightarrow{1} \{(C, 1, \$70)\}
\end{aligned}$$

According to the model definition, these four bids cannot be satisfied together since there is no bid that requests item D. However, in fact these bids are satisfiable since the sum of the bid payoffs $(220 + 380 + 70) - (230 + 350 + 60 + 20) = 10$ is greater than zero and all the bidders get the items or the amount of money they want. This issue can be resolved by introducing a dummy bid (b_d) in order to buy *all* units of *all* items for the price zero:

$$b_d = \{(MONEY, u_d, \$0)\} \xrightarrow{u_d} \{(r_2, u_2'', \$0), (r_3, u_3'', \$0), \ldots, (r_n, u_n'', \$0)\}$$

where u''_j is the number of available units of item r_j ($u''_j = \sum_{i=1}^{m} o_{ij}$ ($j = 2, 3, \ldots, n$)) and u_d is the number of available units of all items in the market ($u_d = \sum_{i=1}^{m} \sum_{j=2}^{n} o_{ij}$). This dummy bid can be automatically entered by the auctioneer after all the bids have been collected from the bidders. After solving the MUDAB problem, either the auctioneer could keep the profit as well as the unrequested items given in the winning bids (item D in the above example) or alternatively let the owners of the winning bids with the unrequested items get what they want without giving any item (e.g. in the above example, bidder 4 could be allowed to get item C without giving item D).

6 Solution Procedure

In this section, we will introduce an algorithm for transforming the winner determination problem of the MUDAB model to a minimum cost circulation flow problem [17]. Let $N(V, A, l, u, c, b)$ denote a network with node set V, arc set A, lower bound $l(v, w)$, capacity $u(v, w)$, cost $c(v, w)$ for each arc $(v, w) \in A$, and supply/demand value $b(v)$ for each node $v \in V$. The minimum cost circulation network can be constructed as follows:

- The set of nodes, V, consists of three types of nodes:

  (i) Item nodes r_j for each item $r_j \in R$ including the special item $MONEY$.

  (ii) Ownership nodes o_{ij} for each bidder $c_i \in C$ and for each item $r_j \in R$ if $o_{ij} > 0$.

  (iii) Split bid nodes b'_k and b''_k for each bid $b_k \in B$.

- The set of arcs, A, consists of four types of arcs:

  (i) An arc $<r_j, o_{ij}>$ for each bidder $c_i \in C$ and for each item $r_j \in R$ if $o_{ij} > 0$. The capacity of the arc is equal to o_{ij} and the cost is equal to 0.

  (ii) An arc $<o_{ij}, b'_k>$ for each bidder $c_i \in C$, for each item $r_j \in R$ and for each bid $b_k = (A_k, u_k, R_k) \in B_i$ if there exists a t such that $(i'_t, u'_t, p'_t) \in A_k$ and $i'_t = j$. The capacity is equal to u'_t and the cost is equal to p'_t.

  (iii) An arc $<b'_k, b''_k>$ for each bid $b_k = (A_k, u_k, R_k) \in B$ with the capacity equal to u_k and the cost equal to 0.

(*iv*) An arc <b''_k, r_j> for each bid $b_k = (A_k, u_k, R_k) \in B$ and for each item $r_j \in R$ if there exists a v such that $(i''_v, u''_v, p''_v) \in R_k$ and $i''_v = j$. The capacity of the arc is equal to u''_v and the cost is equal to $-p''_v$.

There is neither supply nor demand for any node in the network, and therefore $b(v) = 0$ for every node $v \in V$. Also, lower bounds $l(v, w)$ for all arcs $(v, w) \in A$ are set to 0. The minimum cost circulation network for the example problem given in Section 4 and its solution can be seen in Figures 4a and 4b respectively.

The correctness of the network can be verified as follows. The integer variables x'_{kt} are represented with the arcs of type (ii) and x''_{kv} are represented with the arcs of type (iv). The costs of the type (ii) arcs are the reservation prices of the items declared in the ask set whereas the costs of the type (iv) arcs are the additive inverses of the reservation prices of the items declared in the request set. Since the costs of the other arcs are zero, finding the minimum cost circulation flow means optimizing the objective function in Eq.(1). The constraints in Eq.(2) and Eq.(3) are applied by the flow conservation property of the network for the nodes of type (iii) and the nodes of type (i) respectively. The upper limits on the arcs of type (i) ensure that no bidder can give more than the number of units of items he owns which is stated in Eq.(4). Finally, the upper limits on the arcs of type (ii) and (iv) apply the item-level upper exchange limit constraints and the upper limits on the arcs of type (iii) apply the bid-level upper exchange limit constraints.

There are strongly polynomial algorithms for solving minimum cost network flow problems such as Goldberg and Tarjan's minimum mean cycle-canceling algorithm with time complexity $O(|V|^2|A|^3 log|V|)$ [24] and the enhanced capacity scaling algorithm with time complexity $O((|A|log|V|)(|A| + |V|log|V|))$ [17]. As will be shown in the next section, these problems can be solved very fast using specialized solvers such as CPLEX [25] and CS2 [26].

7 Experimental Results

We have prepared test cases for observing the performance and the scalability of the MUDAB model in practice. For solving these cases, we use CS2 software which contains an implementation of scaling push-relabel algorithm for the minimum cost flow problems [26, 27]. Tests are conducted on a dedicated 64 bit Intel Xeon 2.66 GHz workstation with 8 gigabytes of memory using single core.

The test cases are generated using 12 different configurations for simulating different market sizes. In these configurations, the number of bidders varies between 10,000 and 100,000, and the number of bids varies between 100,000 and 1,000,000. The sizes of the request sets (excluding the special item MONEY) are drawn uniformly from the intervals $[0, 2]$, $[0, 10]$ and $[0, 20]$ which are denoted as *small*, *medium* and *large* sizes respectively. Each bidder is assumed to have an average of five different items and up to 100 units of each of these items. The maximum size of an ask set is limited to the number of items of the bid owner. The probability of the bidder to include the special item MONEY inside the ask or request sets is set to 0.25. The rest of the parameters of the test case generator, the generated test cases and their solutions can be obtained from [28].

The results of the experiment are presented in Table 3. For each of the 12 configurations, 20 test cases are generated and the average number of items, the average number of nodes and edges in the network, the average execution time of the network solver and the corresponding standard deviation are reported. The optimum solution to the problem instances containing up to 200,000 bids can be solved within a minute. Even the larger instances with 1,000,000 bids and network size of approximately 3,000,000 nodes and 15,000,000 edges are solvable in less than five minutes demonstrating the scalability of the model in practice.

8 Discussion and Conclusion

Recent advances in information and communication technologies provided us a platform so that we now have:

- high computational power making it possible to solve large scale problems pretty quickly,
- the Internet providing a worldwide communication medium for the people, computers and mobile devices, and
- the World Wide Web providing a high level user interface for the users worldwide.

This platform provides most of the components needed for building large-scale e-marketplaces in which participants worldwide can use in order to engage in trade transactions. Other important components that are needed are market models and algorithmic engines that analyze the bids of the users and extract from the collection of bids, feasible trading patterns that optimize some criteria such as the trading volume and/or the social welfare.

In this paper, we contribute a market model and an algorithm that improve DA based e-marketplaces by allowing barter bids. The MUDAB model allows traders to declare both direct and differential barter requests along with the usual sale and purchase requests. It also extends the previous DAB model which allows bartering of only single items. The MUDAB model can be used virtually in every market in which DA rules are applicable. Furthermore, the MUDAB model utilizes a more powerful bidding language which improves the market outcome when compared with the DAB model. Since the MUDAB model is the superset of both multi-unit DA and the DAB models, the allocative efficiency of the MUDAB model is also guaranteed to be better or at least as good as that of these models. Even with these additional features, the MUDAB market can be cleared fast using the proposed polynomial-time network flow based algorithm. Test instances involving millions of bids can be solved within minutes which opens the way for building realistic e-markets for massive number of users. A drawback of our model and the corresponding algorithm is that it does not allow package bidding, and therefore, the item exchanges are always one-to-one. Even though our model can be generalized to many-to-many exchange option, this will make the problem computationally harder, i.e. NP-Hard, as proven in [18].

The implementation of the model is also straightforward; bids are collected from the bidders within a predefined time period. Then, the optimum allocation is found by running our algorithm and the results are published. Unsatisfied bids could be transferred to the next bidding round unless they are withdrawn. The length of the rounds can be determined according to the rate of submission of bids. For instance, for the described e-media market the length of a round could be as short as 30 minutes whereas for a property exchange the length could be as long as 30 days.

The benefits of bartering mechanism have already been demonstrated in real life. For instance, in the United States, there are approximately 400 barter exchange companies and it is estimated that over 350,000 businesses are involved in bartering activities. According to statistics published by The International Reciprocal Trade Association, $8.25 billion worth of goods and services were traded through barter exchange companies in 2004 [29]. Besides the barter exchanges, there are also non-profit and commercial online platforms that provide bartering services. For instance, swaptree.com offers their members to barter books, music, DVDs or video games. The method of bartering used is the direct bartering without involvement of money. Similarly, rehashclothes.com provides a platform for bartering clothing and goswap.org does the same for bartering houses and land. An example for non-profit platforms would be textswap.com which provides textbook exchange services for college students.

These examples motivate us to envision future e-markets incorporating the differential barter mechanism for increasing the throughput of the real life markets. The known difficulty of finding the double coincidence of wants in bartering [30] is resolved by the algorithmic engines that facilitate multilateral (multi-way) bartering among two or more parties. Furthermore, we believe that the introduced features of the model and the web-based access to the markets will also ease the transition from otherwise producer-consumer oriented markets to the markets in which consumers become traders who buy and sell commodities in order to make profit just like in eBay. This transition will enrich the markets greatly as it has been seen in stock exchanges.

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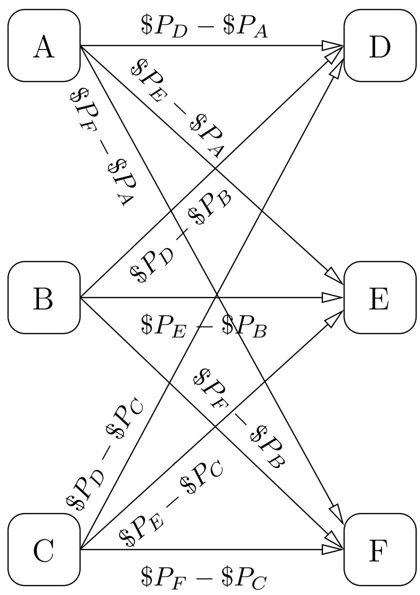


Figure 1a: Scheme of Expressing Differential Prices for Each Pair of Items. This scheme requires $|Ask\ Set| * |Request\ Set| = 9$ prices to be declared.

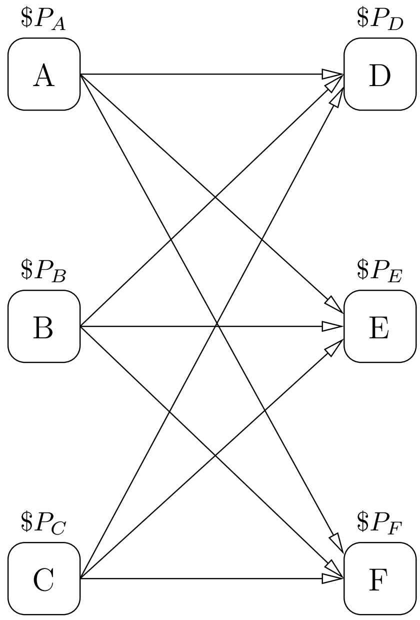


Figure 1b: Scheme of Expressing Differential Prices as Bidder Specified Prices on Items. This scheme requires $|Ask\ Set| + |Request\ Set| = 6$ prices to be declared.

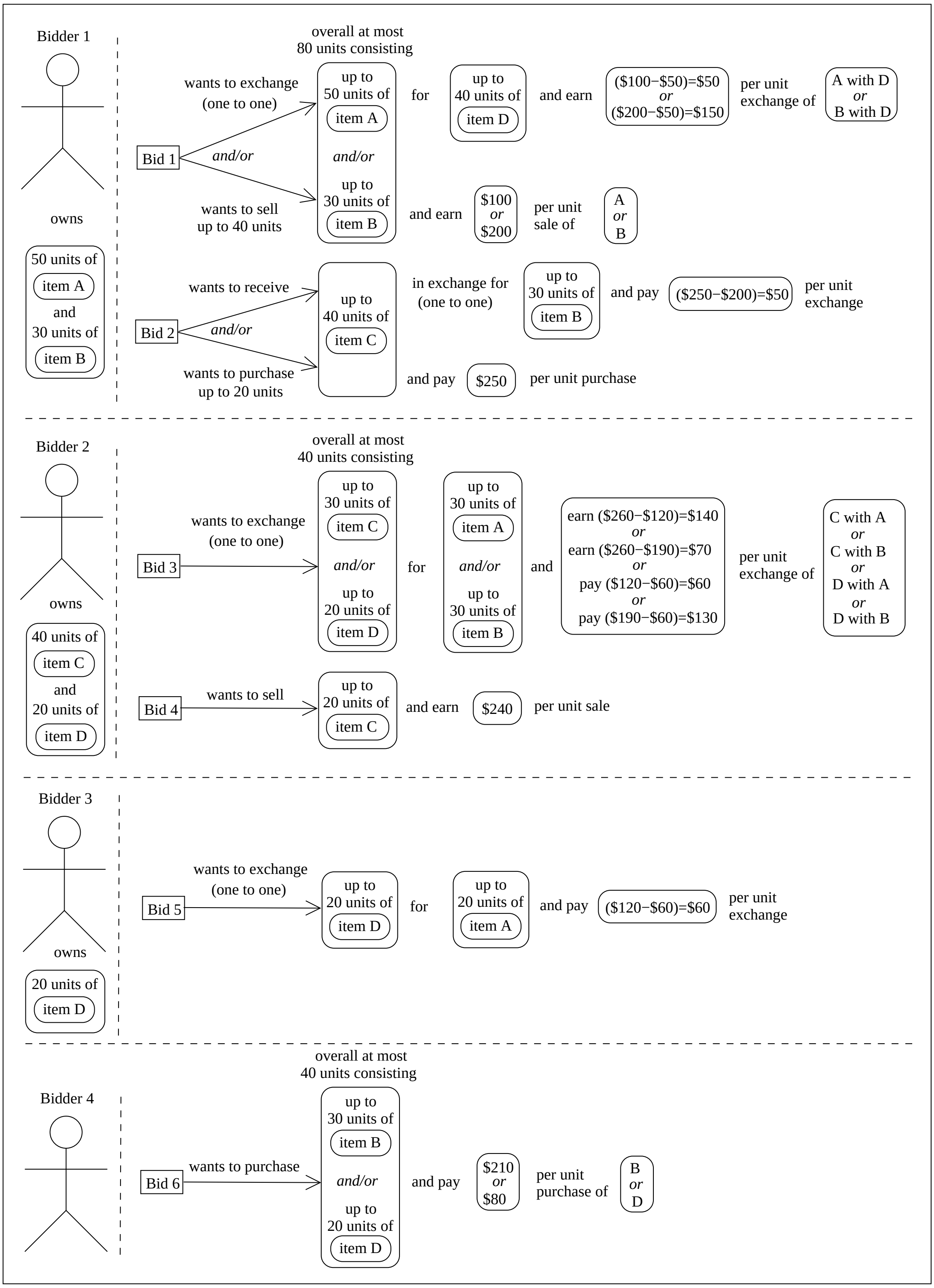


Figure 2: Example Market Scenario for Illustrating the MUDAB Model. Note that the model prevents bidders to give more than the items they own even if they bid so.

Ownership Table:

	Item A	Item B	Item C	Item D
Bidder 1	50 units	30 units	-	-
Bidder 2	-	-	40 units	20 units
Bidder 3	-	-	-	20 units
Bidder 4	-	-	-	-

Bids:

Bid Representation:

$$\underbrace{\{\texttt{(item, exchange limit, price)},\ldots\}}_{Ask\ \ Set} \xrightarrow[\texttt{limit}]{\texttt{bid level}} \underbrace{\{\texttt{(item, exchange limit, price)},\ldots\}}_{Request\ \ Set}$$

Bidder 1

Bid 1 $:\{(A,50,\$100),(B,30,\$200)\} \xrightarrow{80} \{(D,40,\$50),(MONEY,40,\$0)\}$

Bid 2 $:\{(B,30,\$200),(MONEY,20,\$0)\} \xrightarrow{40} \{(C,40,\$250)\}$

Bidder 2

Bid 3 $:\{(C,30,\$260),(D,20,\$60)\} \xrightarrow{40} \{(A,30,\$120),(B,30,\$190)\}$

Bid 4 $:\{(C,20,\$240)\} \xrightarrow{20} \{(MONEY,20,\$0)\}$

Bidder 3

Bid 5 $:\{(D,20,\$60)\} \xrightarrow{20} \{(A,20,\$120)\}$

Bidder 4

Bid 6 $:\{(MONEY,40,\$0)\} \xrightarrow{40} \{(B,30,\$210),(D,20,\$80)\}$

Figure 3: Representation of the Example Market Scenario in Figure 2. The scenario is represented using the new bidding language of the MUDAB model.

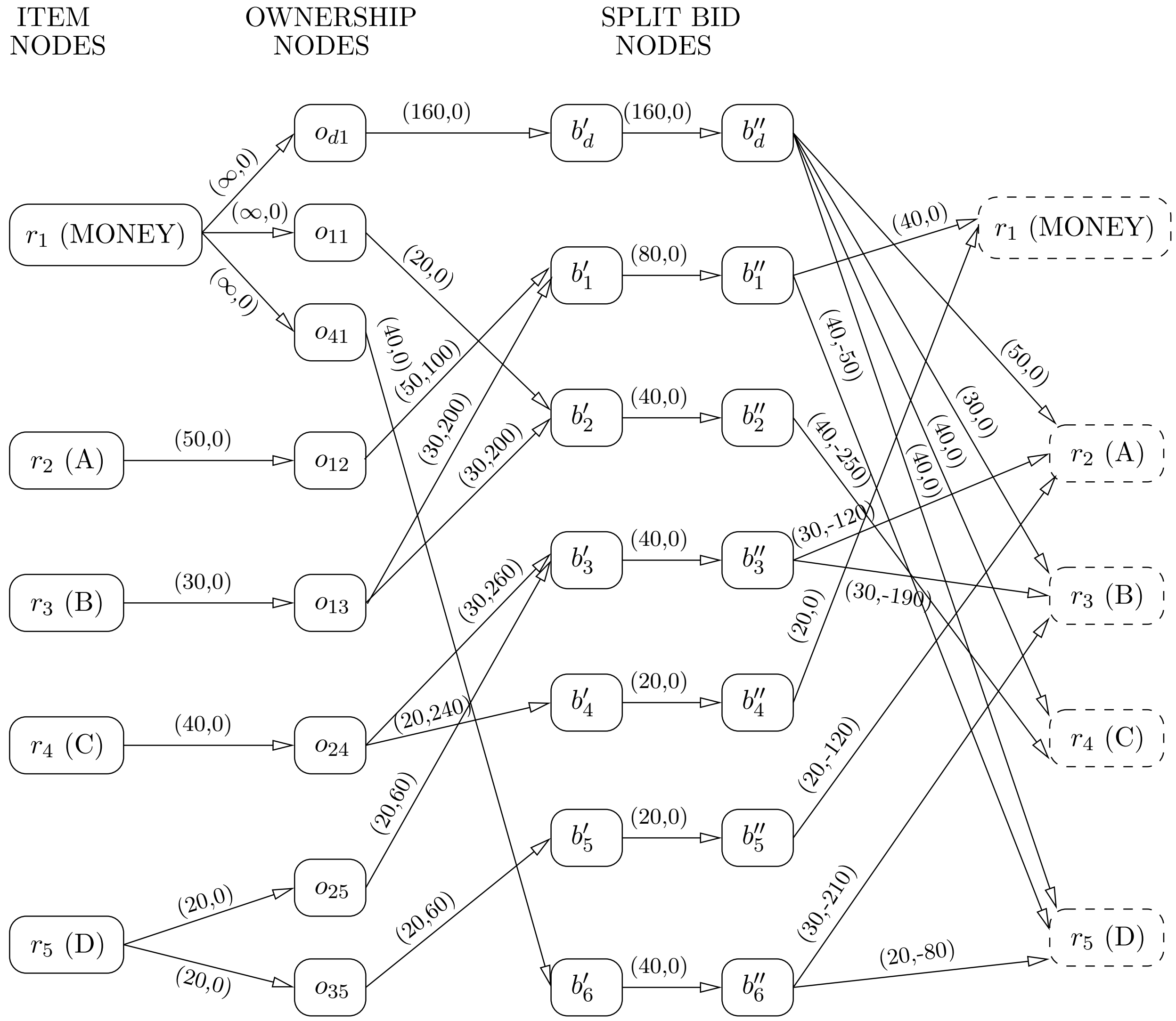
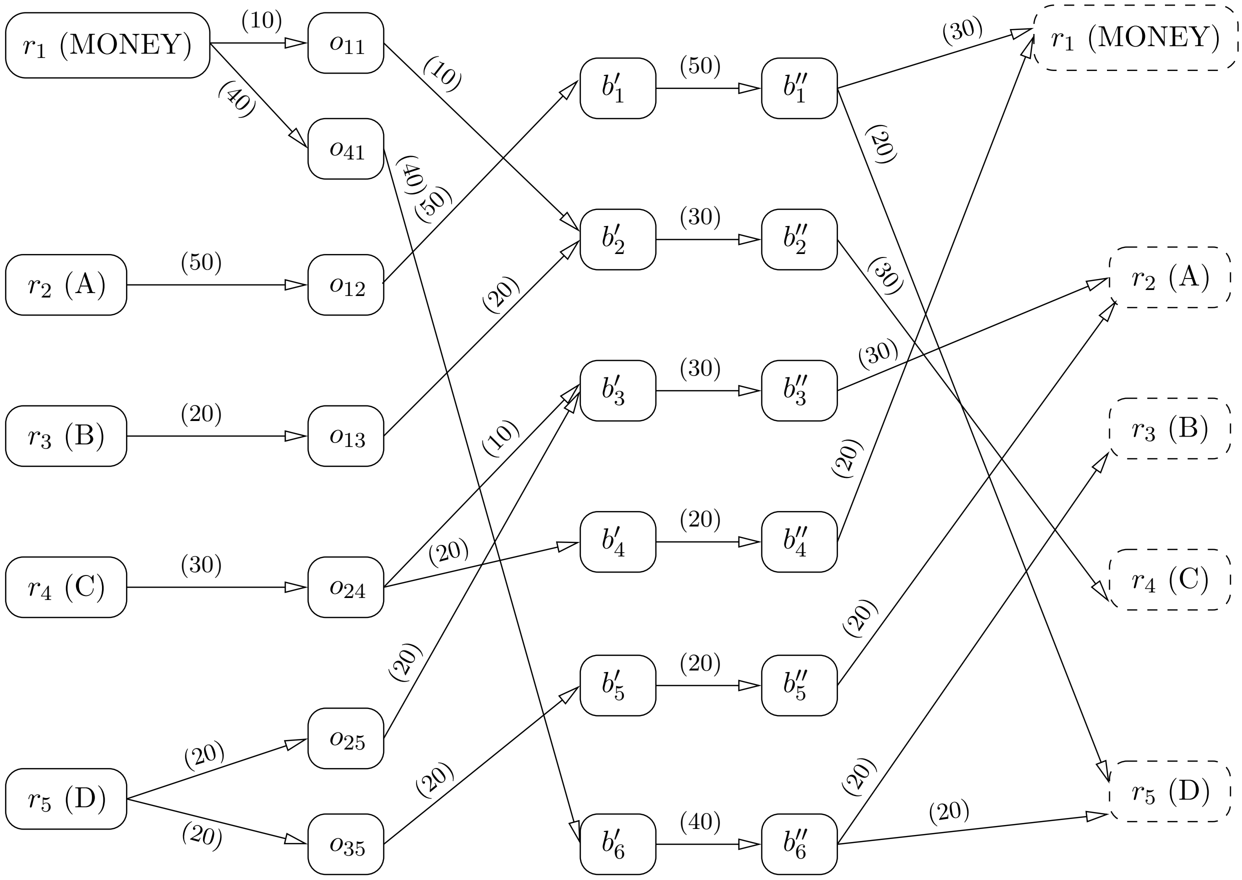


Figure 4a: Minimum Cost Circulation Network of the Example in Section 4. (Capacity,Cost) values are presented on the arcs. Note that the dashed nodes located on the right side are the same nodes located on the left side.



Bid 1: Bidder 1 exchanges 20 units of item A for 20 units of item D, and sells 30 units of item A. He earns \$4000.

Bid 2: Bidder 1 exchanges 20 units of item B for 20 units of item C, and buys 10 units of item C. He pays \$3500.

Bid 3: Bidder 2 exchanges 10 units of item C and 20 units of item D for 30 units of item A. He earns \$200.

Bid 4: Bidder 2 sells 20 units of item C. He earns \$4800.

Bid 5: Bidder 3 exchanges 20 units of item D for 20 units of item A. He pays \$1200.

Bid 6: Bidder 4 buys 20 units of item B and 20 units of item D. He pays \$5800.

Figure 4b: Solution of the Minimum Cost Circulation Network in Figure 4a. (Flow) values are presented on the arcs.

	owns	*wants*
Bidder 1	200 rolls of grade B paper and 100 rolls of grade C paper	to sell 200 rolls of his inventory
Bidder 2	100 rolls of grade B paper	to exchange his inventory for 100 rolls of grade C paper
Bidder 3	100 rolls of grade A paper	to exchange his inventory for 100 rolls of grade B paper
Bidder 4	-	to buy 100 rolls of grade A paper
Bidder 5	-	to buy 100 rolls of grade B paper

Table 1a: An Example Scenario for Paper Auction. The second column indicates the inventories of the bidders before the auction and the third column indicates the requests of the bidders.

	Grade A	Grade B	Grade C
Bidder 1	-	$300	$220
Bidder 2	-	$310	$210
Bidder 3	$400	$290	-
Bidder 4	$430	-	-
Bidder 5	-	$300	-

Table 1b: Reservation Prices of the Bidders for the Example Scenario in Table 1a. The prices are declared per roll of paper.

	Number of Units Traded	*Number of Required Bids*	*Problem Size*	*General Market Clearing Algo. Complexity*
Single-Unit DA	100	400	n = 7 (bids)	$O(n \log n)$
Multi-Unit DA	100	3	n = 7 (bids)	$O(n \log n)$
DAB	100	40,600	n = 1,302 (nodes) m = 41,200 (arcs)	$O(n^2 m^3 (\log n))$
MUDAB	400	5	n = 23 (nodes) m = 35 (arcs)	$O(n^2 m^3 (\log n))$

Table 1c: Comparison of the Auction Models. The outcomes of the single-unit DA, the multi-unit DA, the DAB, and the MUDAB models are presented for the example scenario in Table 1a.

Taxonomy of Barter Models

Allowed Item Exchange Type	Identical Units of Items in the Market: *Single-Unit*	*Multi-Unit*
Single-Item *(one-to-one)*	The DAB Model in (Özturan 2005) *(Polynomial)*	The MUDAB Model in This Paper *(Polynomial)*
Multi-Item *(package bidding)*	The Single-Unit Multi Resource Bartering Problem in (Özturan 2004) *(NP-Hard)*	The Multi-Unit Multi Resource Bartering Problem in (Özturan 2004) *(NP-Hard)*

Table 2: Taxonomy of Barter Models. Classification is based on whether identical units of items are allowed in the market and whether the bidding language allows package (combinatorial) bidding.

Table 3: Average Network Sizes and Execution Times of the Network Solver for Test Cases. Each test case configuration consists of 20 instances.

	Request			*Execution Time (seconds)*	
Bids	*Set Size*	*Nodes*	*Edges*	*mean*	*stdev*
	small	310,086	639,944	9.19	0.58
100,000	medium	310,039	1,018,887	12.36	0.61
	large	310,017	1,514,825	15.99	0.82
	small	619,956	1,278,457	21.99	1.39
200,000	medium	620,099	2,038,985	29.77	1.85
	large	620,006	3,029,529	38.47	1.41
	small	1,549,931	3,196,936	64.15	4.15
500,000	medium	1,549,876	5,092,334	83.57	4.97
	large	1,550,019	7,573,373	108.39	3.21
	small	3,099,875	6,393,790	137.03	10.80
1,000,000	medium	3,100,030	10,188,917	193.61	6.05
	large	3,100,136	15,150,880	247.84	14.12